Gauss-Jordan Elimination

Finite Math

23 October 2018

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where m, n, p are real numbers and $p \neq 0$.

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Definition (Reduced Form)

A matrix is in reduced form if

1. *Each row consisting entirely of zeros is below any row having at least one nonzero element.*
2. *The leftmost nonzero element in each row is* 1*.*
3. *All other elements in the column containing the leftmost* 1 *of a given row are zeros.*
4. *The leftmost* 1 *in any row is to the right of the leftmost* 1 *in the row above.*

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$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -3 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 9 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

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$$\left[\begin{array}{cccc|c} 1 & 4 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

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$$\left[\begin{array}{cccc|c} 1 & 4 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 0 & 4 & 8 \\ 0 & 1 & 1 & 9 \\ 0 & 0 & 0 & 2 \end{array}\right]$$

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Why are the following matrices not in reduced form? Put them in reduced form:

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2. $$\left[\begin{array}{ccc|c} 1 & 5 & 4 & 3 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

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3. $\left[\begin{array}{ccc|c} 0 & 1 & 0 & -3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]$

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3. $\left[\begin{array}{ccc|c} 0 & 1 & 0 & -3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]$

4. $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{array}\right]$

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Solve the following system using Gauss-Jordan elimination:

$$\begin{array}{rcrcrcl} 3x & + & y & - & 2z & = & 2 \\ x & - & 2y & + & z & = & 3 \\ 2x & - & y & - & 3z & = & 3 \end{array}$$

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Solve by Gauss-Jordan elimination:

$$\begin{array}{rcrcrcr} 2x_1 & - & 4x_2 & - & x_3 & = & -8 \\ 4x_1 & - & 8x_2 & + & 3x_3 & = & 4 \\ -2x_1 & + & 4x_2 & + & x_3 & = & 11 \end{array}$$

Now You Try It!

Example

Solve by Gauss-Jordan elimination:

1. $$\begin{aligned} 3x_1 + 5x_2 - x_3 &= -7 \\ x_1 + x_2 + x_3 &= -1 \\ 2x_1 + 11x_3 &= 7 \end{aligned}$$

2. $$\begin{aligned} 3x_1 - 4x_2 - x_3 &= 1 \\ 2x_1 - 3x_2 + x_3 &= 1 \\ x_1 - 2x_2 + 3x_3 &= 2 \end{aligned}$$

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Solution

(1) $x_1 = -2, x_2 = 0, x_3 = 1$*, (2) No solution.*